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On Fermi Quantum Fields Constructed from Bose Quantum Fields and Their Applications

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Abstract

In former papers a representation of the quantum Fermi and para-Fermi fields was proposed. This representation is such that the only basic quantum entities are Bose quantum fields. In this paper we show several possibilities of application: (i) to lower the number of "elementary" particles; (ii) to describe as separate states of a fundamental particle other particles that presently are considered as different, and to induce an ordering among them; (iii) to obtain relations among the quantum numbers of those particles; (iv) to obtain a physical picture of some unstable particles. This article is concerned with the physical interpretation of the formalism, and some of the statements that are contained here have a conjectural character.

1. *Introduction*

In a previous paper (Kálnay, 1975), hereafter called (A), we simplified and generalized the paper Kálnay et al. (1973), where it was shown that quantum Fermi (and para-Fermi) fields can be described in terms of quantum Bose fields plus certain c-number coefficients. Briefly, we shall call it the *Bose representation of Fermions.* It is well understood, however, that the c-number coefficients can be compacted into entities that can be interpreted as classical (in the sense of c-number) fields of mixed spinor-tensor indices. See Section 2 of paper (A). In a subsequent paper (Kálnay and Mac Cotrina, 1976), hereafter called (B), we retrieved more orthodox expressions of the Bose described Fermi physical variables, an orthodox form that was lost in the previous work. [Cf. (B) and the Appendix of Kálnay et al. (1973).] Physical variables are discussed in Kálnay and Kademova (1975a) and the transformation laws in Kálnay and Kademova (1975b). A specific model is given in Kálnay (1977) and a review and discussion in Kálnay (1978).

Although not claiming full mathematical rigor, we consider that the results of the above-mentioned papers stand on a reasonably sound basis. They mainly consist of theorems proved according to the standards of theoretical physics. The situation may be quite different as regards the physical interpretation of the Bose representation of fermions, since it partially involves conjectures and plausibility arguments. That is why we separated the physical interpretation and the discussion of possible applications of the Bose representation of fermions from the development of the formalism. Papers (A) and (B) were devoted to such developments, whereas the present article discusses the physical possibilities that the formalism suggests. We did this in order to ensure the validity of papers (A) and (B) in case experiments disprove the physical assumptions herewith expressed. In that case, the physical interpretation of the Bose representation of fermions could be improved, without invalidating the formalism of papers (A) and (B).

We have hopes that some of the ideas we shall express can be fruitfully applied to any of the branches of physics (such as *solid state* or *nuclear physics*) that use quantum theory fields. When we speak of elementary particles, we shall refer to the "elementary" particles of any specific branch of physics, unless the contrary is explicitly stated. However, we believe that the most fundamental applications of the Bose representation of fermions can be done in high-energy physics.

Each of the following sections will be devoted to a different physical aspect of the theory, though some of those aspects have some mutual relationships. Different degrees of plausibility will be found in these parts of the paper.

2. *On the Total Number of "Elementary" Particles*

2.1 *Fermions.* In Kálnay *et al.* (1973)[1] and in Section 3 of (A)[2] we proved that there exist functions of the Bose quantum fields that have all the properties of the quantum Fermi annihilation and creation operators when they act on the one-boson subspace $\mathscr{B}_1$: We reproduce equation (1.5a) of (A) for the Fermi annihilation field $f_\xi(\mathbf{z})$[3]:

$$f_\xi(\mathbf{z}) = \int d^3x \int d^3x' \sum_{\zeta\zeta'=1}^{R} F_{\xi\zeta\zeta'}(\mathbf{z}, \mathbf{x}, \mathbf{x}') b_\zeta^+(\mathbf{x}) b_{\zeta'}(\mathbf{x}') \tag{2.1a}$$

and from equation (1.5b) that of the Fermi creation field

$$f_\xi^+(\mathbf{z}) = \int d^3x \int d^3x' \sum_{\zeta\zeta'=1}^{R} F_{\xi\zeta\zeta'}^+(\mathbf{z}, \mathbf{x}, \mathbf{x}') b_\zeta^+(\mathbf{x}) b_{\zeta'}(\mathbf{x}') \tag{2.1b}$$

The subspace $\mathscr{B}_1$ is the Bose representation of the space $\mathscr{F}^1$ of all possible Fermi states. This is the Bose representation of fermions. As the mathematics of the Bose representation of fermions is isomorphic to that of the standard

[1] Fock representation of the Fermi (and para-Fermi) commutation relations.
[2] Fock and non-Fock cases.
[3] We use the same notation as in papers (A) and (B).

quantum theory of fermions, no mathematical or physical property of fermions can be lost. For example, we have shown explicitly in Kálnay et al. (1973) and in paper (A) that the Pauli principle is satisfied by the Bose constructed fermions in spite of not being satisfied by the underlying Bose field.

We have stated Kálnay et al., 1973[1]; Section 3 of paper (A)[2] that the standard quantum field theory of fermions is isomorphic to the Bose representation of fermions. Then both theories can be equivalently used in physics. However, in the second theory no quantum Fermi field is needed in the set of the elementary particles if quantum Bose fields are available. [Suitable sets of c numbers $F_{\xi\zeta\zeta'}(\mathbf{z}, \mathbf{x}, \mathbf{x}')$ are needed, see Section 2 of (A).] As in a quantum field theory, the fundamental particles are represented by quantum creation and annihilation fields. As no Fermi fields are needed, it is clear that in a theory that contains quantum Bose and Fermi systems the Fermi systems must not be considered as elementary ones. These are derived in terms of the quantum bosons and of the c numbers $F_{\xi\zeta\zeta'}(\mathbf{z}, \mathbf{x}, \mathbf{x}')$. We conclude that *quantum fermions can be ruled out as microscopic elementary particles.*

Remark 2.1.1. In the appendix of Kálnay et al. (1973) we have shown how to construct, in terms of a quantum Bose field, a Bose Hamiltonian

$$
\begin{aligned}
H_{\mathscr{B}}(b, b^+) = \int d^3z \int d^3x \int d^3x' \int d^3x'' \int d^3x''' \sum_{\xi\xi'=1}^{4} \sum_{\zeta\zeta'\zeta''\zeta'''=1}^{R} F^+_{\xi\zeta\zeta'}(\mathbf{z}, \mathbf{x}, \mathbf{x}') \\
\times (-i\nabla_x \cdot \boldsymbol{\alpha} + m\beta)_{\xi\xi'} F_{\xi'\zeta'''\zeta''}(\mathbf{z}, \mathbf{x}'', \mathbf{x}''') \\
\times\, b^+_{\zeta}(\mathbf{x}) b_{\zeta'}(\mathbf{x}') b_{\zeta''}(\mathbf{x}'') b^+_{\zeta'''}(\mathbf{x}''')
\end{aligned}
\tag{2.2}
$$

such that the Bose constructed Fermi field evolves in time according to Dirac's equation for the electron. [In Kálnay and Kademova (1975a) this procedure was extended for arbitrary physical variables. In paper (B) the form of those variables was simplified.] The difficulty is that (2.2) has not the form of a standard Bose Hamiltonian. It could be suspected that the Bose Hamiltonian (2.2) needed for the time evolution of the Fermi field could be inconsistent with the original time evolution of the Bose field. The answer to the problem is derived from the study of the physical variables of bosons and fermions given in Kálnay and Kademova (1975a) and summarized here. In order to fix ideas we shall consider as a first example (cf. Kálnay and Kademova 1975a) the Dirac electron represented in Bose terms as in the Appendix of Kálnay et al. (1973). Let us call $H_{\mathscr{B}}$ the Bose representation (2.2) of the Hamiltonian of the quantum electron, and $H^{\mathscr{B}}$ the Hamiltonian of the original quantum Bose field. There is no conflict between the time evolutions of bosons and fermions because *there is no need for $H_{\mathscr{B}}$ to equal $H^{\mathscr{B}}$.* In fact, we have shown in Section 2 of (A) that the c number $F_{\xi\zeta\zeta'}(\mathbf{z}, \mathbf{x}, \mathbf{x}')$ used, in addition to the quantum Bose fields to represent the quantum Fermi fields [equation (2.1a)], can be interpreted as classical fields; and *as such, they can (or perhaps, they should) carry one part of the energy.* On the other hand, there may be an interaction energy between the quantum bosons and the classical fields, which is a further reason for the total energy $H_{\mathscr{B}}$ being different from the energy $H^{\mathscr{B}}$

of the Bose system. Then generally $H_{\mathscr{B}} \neq H^{\mathscr{B}}$ but, whatever the time evolution of the quantum Bose field $b_\zeta(\mathrm{x})$ in equation (2.2), the time evolution given by Dirac's equation is obtained for the quantum electron (see the proof in Kálnay and Kademova, 1975a, b).

Remark 2.1.2. We have a similar situation concerning angular momentum (cf. Kálnay and Kademova, 1975a, b). Let us consider equation (2.1a): The Bose field $b_\zeta(\mathrm{x})$ carrying integer spin belongs to a tensor representation of the rotation group. (The tensor index ζ may be a set of indices $\zeta_1, \zeta_2, \ldots, \zeta_a$.) However, the tensor indices ζ, ζ' in equation (2.1a) are contracted with the tensor indices ζ, ζ' of classical field $F_{\xi\zeta\zeta'}(\mathrm{z}, \mathrm{x}, \mathrm{x}')$ so that finally the quantum Fermi field $f_\xi(\mathrm{z})$ transforms according to the ξ dependence of the classical field $F_{\xi\zeta\zeta'}(\mathrm{z}, \mathrm{x}, \mathrm{x}')$. It is sufficient that the classical field has ζ and ζ' as suitable tensor and ξ spinor indices for having the quantum Fermi field $f_\xi(\mathrm{z})$ transformed according to a spinor representation of the rotation group[4] This means that the quantum generators of rotations (i.e., the angular momentum operators) of the quantum Bose and Fermi fields are not equal. This is the correct result: The Bose representation $\mathbf{j}_{\mathscr{B}}(b, b^+)$ of the angular momentum of the fermion cannot coincide with the angular momentum $\mathbf{j}^{\mathscr{B}}(b, b^+)$ of the boson because the fermion has half-integral eigenvalues while those of the latter are always integral. Within this scheme of ideas the problem is considered in Kálnay and Kademova (1975b). Incidentally, notice that the classical field has its own transformation laws under rotation, which means that it carries spin. This adds further support to the idea that the classical field is a physical entity. They are carriers of angular momentum, and (cf. previous Remark) of energy.

Remark 2.1.3. We have seen that from the Bose representation of fermions developed in the previous papers it results that quantum fermions can be ruled out as microscopic elementary particles. If the point of view [we call it (i)] that the $F_{\xi\zeta\zeta'}(\mathrm{z}, \mathrm{x}, \mathrm{x}')$ be only c-number coefficients in the construction of wave packets is adopted [see Section 2 of (A)], then the only fundamental entities are the quantum bosons. If, on the other hand [point of view (ii)], one considers the $F_{\xi\zeta\zeta'}(\mathrm{z}, \mathrm{x}, \mathrm{x}')$ as classical fields [see again Section 2 of (A)], then it could be said that the quantum fermions are constructed in terms of quantum bosons and classical fields. In both cases the quantum fermions can be expressed in terms of more elementary entities. As the Bose representation of fermions is shown by theorems proven according to the standards of theoretical physics (see references), *the statement that quantum fermions can always be described as nonelementary entities has a sound basis*. This certainty contrasts with that of other statements we shall make in the remainder of this paper, supported by less conclusive arguments. To begin with, if the point of view (ii) is correct, then one should expect that some future experiment could detect those classical fields. One could *conjecture*, although it would *not* be essential, that those classical fields correspond to some up-to-

[4] We remark on the strange fact that by appropriately changing the $F_{\xi\zeta\zeta'}(\mathrm{z}, \mathrm{x}, \mathrm{x}')$, the Bose field $b_\zeta(\mathrm{x})$ may be changed without modifying the quantum Fermi system.

now undetected (because of the lack of a suitable interaction knowledge) macroscopic field.

2.2 *Parafermions.* Parafields are the fields obeying Green's (1953) commutation relations. Among the work done on parafields we would like to quote the paper by Greenberg and Messiah (1965). Several models of physical system parafields have been used as in the quark model (Greenberg, 1964), the nuclear pairing force model of a single j shell (Cusson, 1969), and the spin-$\frac{1}{2}$ oscillators with spin-orbit interaction (Cusson, 1969). However, the existence of paraparticles among the *known* fundamental particles of high-energy physics was questioned because of some selection rules that would not allow for their existence in nature. See, e.g. Greenberg and Messiah (1965). However, these selection rules strongly depend upon some assumed hypotheses, which (even if likely) may not be right (Feshbach and Tomljanovich, 1967). Moreover, for most particles no direct experimental determination of the statistics has been made (Feshbach and Tomljanovich, 1967; Perkins, 1972b). Therefore, *we shall not exclude the possibility that some of the observed "elementary" particles of high-energy physics are parafermions.* Other authors also consider parafields in elementary particle physics, for example Green (1972) and Scharfstein (1972, 1973).

For at least the majority of "elementary" particles in high-energy physics, there has been no publicized experiment performed that involves a number of particles to give a *direct* knowledge of statistics. The statistics are generally obtained by measuring spin and using the connection between spin and statistics. However, if we consider for example a spin-$\frac{1}{2}$ particle, it only follows that the particle is a fermion *if* boson and fermion are the only allowed cases. As a matter of fact, that connection does not state that a spin-$\frac{1}{2}$ particle must be a fermion. It only says that a spin-$\frac{1}{2}$ particle cannot be a boson. Thus, there is, in principle, place for the paraparticles.

Perkins (1972b) has shown that even with the photon that is the boson for which no example concerning statistics is available, the statistics determination is not clear. In fact, "the only direct evidence of the statistics of the photon (black-body-radiation experiments) can be satisfied" (Perkins, 1972b) for at least two different statistics. Most specifically, Perkins (1972a) has shown that there exists at least one new statistics (which is not Bose and perhaps not even para) such that the photon distribution "is similar enough to Planck's distribution to satisfy experimental results" (Perkins, 1972a) within the experimental error. So that even for the photon we are not completely sure of the statistics. And for the photon we certainly have a lot of experiments involving many particles! If even the statistics of the photon was not definitely proven by experiments, why then exclude the possibility that some resonances might be of paraparticles?

On the other hand, many-boson states certainly are observed in nature, and, because of the work done in Kálnay et al. (1973) and in paper (A),[5] we

[5] We shall assume in what follows a Fock representation of the Fermi and para-Fermi commutation relation; cf. Section 4 of (A).

deduce that they can be differently described as para-Fermi states. [See Section 4 of (A).] As a consequence, we deduce from the formalism of the paper by Kálnay et al. (1973) and of (A) that *para-Fermi states exist in nature.*

Going back to high-energy physics, we shall assume that [paying if necessary the price that some likely condition such as paralocality (Greenberg and Messiah, 1965) may be violated] some of the observed "elementary" particles could, in principle, be parafermions.

Then, in the same form as in Section 2.1, where we found that the Fermi particles can be retired from the set of the true quantum *elementary* particles, we now deduce from the formalism of the previous papers that those "elementary" particles that are parafermions can also be retired from the set of the true quantum *elementary* particles. In fact it is shown [Kálnay et al. 1973; and in Section 4 of paper (A)] that when the Bose constructed fields (2.1) act on the whole Bose state vector space $\mathscr{B}$ they satisfy Green's (1953) commutation relations characteristic of para-Fermi fields. From now on, the argument is similar to that of Section 2.1 of the present paper, and we shall not repeat it here.

Moreover, in paper (A) we gave the necessary and sufficient conditions for obtaining in the subspace $\mathscr{B}_1$ a Fock representation of the Fermi algebra, and it is proven that in that case *the same* fields (2.1) act on the p-boson states subspace $\mathscr{B}_p$ as an irreducible Fock representation of the para-Fermi algebra corresponding to an order p (Green, 1953; Greenberg and Messiah, 1965) of parastatistics. Thus, the para-Fermi particles can also be ruled out as elementary ones if they are additionally required to correspond to a specific order of parastatistics in a Fock representation.

3. *Different States vs. Different Particles*

Let us assume that in nature there exist a fermion A_1 and a parafermion A_p of order p of parastatistics[5] and also a boson B. According to the previous discussion we can rule out A_1 and A_p as true elementary particles describing their quantum fields in terms of the Bose field $b_\zeta(\mathbf{x})$ of B according to equations (1.1). We then represent the state vector space of A_1 by the subspace $\mathscr{B}_1$ of the one-boson states, and the state vector space of A_p by the subspace $\mathscr{B}_p$ of the p-boson states of B.

In the traditional picture of the facts, the particles A_1, A_p, and B are viewed as three different particles. In the Bose description of fermions and parafermions the state of *one* A_1 particle is a linear superposition of states like the (2.6) of (A), i.e. (see footnote 3 above),

$$f_\xi^+(\mathbf{z})|0\rangle^{\mathscr{F}_1} = \sum_{\zeta\zeta'=1}^{R} \int d^3x \int d^3x' F^+_{\xi\zeta\zeta'}(\mathbf{z},\mathbf{x},\mathbf{x}')\mathcal{O}_{\zeta'}(\mathbf{x}')b_\zeta^+(\mathbf{x})|0\rangle^{\mathscr{B}} \in \mathscr{B}_1 \qquad (3.1)$$

the state of *one* A_p particle is a linear superposition of the states obtained by applying the para-fermi creation field (1.1b) to the para-Fermi vacuum [see (4.2) of (A)] i.e., applying (1.1b) to

$$|0\rangle^{\mathscr{F}1} = \int d^3x_1 \cdots \int d^3x_p \sum_{\zeta_1 \cdots \zeta_p = 1}^{R} \mathcal{O}_{\zeta_1}(\mathbf{x}_1) \cdots \mathcal{O}_{\zeta_p}(\mathbf{x}_p) b^+_{\zeta_1}(\mathbf{x}_1) \cdots b^+_{\zeta_p}(\mathbf{x}_p)|0\rangle^{\mathscr{B}} \in \mathscr{B}_p \tag{3.2}$$

it follows that the state of *one* A_p particle is (as the para-Fermi vacuum $|0\rangle^{\mathscr{F}1}$) also a state of $\mathscr{B}_p$; the state of *one* B particle is a linear superposition of

$$b^+_\zeta(\mathbf{x})|0\rangle^{\mathscr{B}} \tag{3.3}$$

the state space of an *arbitrary* number of A_p particles (p fixed) coincides with $\mathscr{B}_p$ and it is spanned by the basis [cf. Kálnay et al., 1973; and paper (A)]

$$(n!)^{-1/2} b^+_{\zeta_1}(\mathbf{x}_1) \cdots b^+_{\zeta_p}(\mathbf{x}_p)|0\rangle^{\mathscr{B}} \in \mathscr{B}_p \tag{3.4}$$

finally, the set of an *arbitrary* number of A_p particles (variable p) coincides with the space

$$\mathscr{B} = \bigoplus_{p=0}^{\infty} \mathscr{B}_p \tag{3.5}$$

of all Bose states of the B boson.

This means that if the experimental conditions are such that if we focus our attention on (i) the one-boson states, then the Bose system allows a description in which it looks like a Fermi system, or (ii) on the p-boson states, then the Bose system allows a description in which it looks like a para-Fermi system of order p of parastatistics, or (iii) on arbitrary Bose states, then the more natural (but not unique) view is to consider it as a Bose system.

We point out that in the same way that the introduction of the isospin allowed physicists to see protons and neutrons as different states of one and the same particle, the Bose description of fermions and parafermions offers the possibility of seeing the particles A_1 and A_p as different states of one and the same particle: a boson.

However, the above idea looks artificial (though logically possible) if only the one-boson and the p-boson states (with fixed p) correspond to existing particles A_1 and A_p. A logically unnecessary, but very natural, prediction of the theory is that there exists in nature a numerable set of particles $A_1, A_2, \ldots, A_p, A_{p+1}, \ldots$ corresponding to the different subspaces $\mathscr{B}_p$ of the Bose system. The increasing number of "elementary" particles discovered in high-energy physics suggests, though it does not prove, that this may be right.

To sum up: *The Bose description of fermions and parafermions offers a possibility of a more unified description of the so-called "elementary particles," even permitting their subsequent classification according to the different Bose subspaces.* Although we are simplifying the discussion of these sections by restricting ourselves to the Fock representations (cf. footnote 5 above), one should also consider non-Fock representations, which allow for greater freedom and possibilities (Govorkov, 1973).

4. *Possibility of Relationships among Quantum Numbers*

Let us consider a Bose description in terms of a boson B of the fermion A_1 and of the parafermions $A_2, A_3, \ldots$ as in the last section. We shall add to the formalism described in papers (A) and (B) an assumption, and shall explore its consequences.

Assumption 4.1. Let us choose a given type of physical variable Ω (e.g., the Hamiltonian). Then the operators $\Omega_{\mathscr{F}^1}, \Omega_{\mathscr{F}^2}, \ldots, \Omega_{\mathscr{F}^p}, \ldots$ that represent this variable Ω for the different particles $A_1, A_2, \ldots, A_p, \ldots$ are all equal when represented in terms of B:

$$\Omega_{\mathscr{B}^1}(b, b^+) = \Omega_{\mathscr{B}^2}(b, b^+) = \cdots \equiv \Omega_{\mathscr{B}}(b, b^+) \tag{4.1}$$

where (Kálnay and Kademova, 1975a)

$$\Omega_{\mathscr{B}^i}(b, b^+) \equiv \Omega_{\mathscr{F}^i}[f(b, b^+), f^+(b, b^+)] \tag{4.2}$$

Now let us suppose that we have sufficient experimental information concerning, e.g., A_3 for having a good candidate for $\Omega_{\mathscr{B}^3}$. If Assumption 4.1 is right, then by looking for the eigenvalues of $\Omega_{\mathscr{B}^3}$ (b, b^+) on the subspaces $\mathscr{B}_1$ and $\mathscr{B}_2$ we can predict the values of Ω for, respectively, the particles A_1 and A_2. (Notice that if conversely one starts from A_1, one is not sure of the operators $\Omega_{\mathscr{B}^2}$ and $\Omega_{\mathscr{B}^3}$ because to a good $\Omega_{\mathscr{B}^1}$ one can always add an operator δ_1 such that $\delta_1\mathscr{B}_1 = 0$, $\delta_1\mathscr{B}_2 \neq 0$, $\delta_1\mathscr{B}_3 \neq 0$. [See, e.g., the operator δ_1 considered in Section 2 of paper (B), which does not alter the eigenvalues of $\Omega_{\mathscr{B}^1}$ in $\mathscr{B}_1$, but which may alter them in $\mathscr{B}_2$ and $\mathscr{B}_3$.]

Thus, Assumption 4.1 offers, if correct, a possibility of predicting eigenvalues of a variable concerning one particle, by using information obtained from another particle.

Moreover, the eigenvalues of an operator in different subspaces are usually related by recurrence formulas (example: the calculations of several energy spectra in elementary quantum mechanics). Then, *the possibility of obtaining relations among eigenvalues arises.* We are thinking on analogs of perhaps the mass formulas. As an example, we computed in Kálnay (1977) a simple model of the Bose representation of fermions: The energy eigenvalues of the parafermions are, in this model, linear combinations of that of the fermions.

As a next step one would be tempted to generalize Assumption 4.1 by imposing that also the Ω operator ($\Omega^{\mathscr{B}}(b, b^+)$ for the B particle) be identical to $\Omega_{\mathscr{B}}(b, b^+)$. This may perhaps happen in certain cases, but not in general. In fact, because of Remark 2.1.1 it would be exceptional to have $\Omega^{\mathscr{B}} = \Omega_{\mathscr{B}}$ when Ω is the energy, and because of Remark 2.1.2 it is impossible to have $\Omega^{\mathscr{B}} = \Omega_{\mathscr{B}}$ when Ω is a component of the angular momentum.

5. *Unstable Particles*

Let us now consider a different possibility: The Hamiltonian $H^{\mathscr{B}}$ of the underlying Bose system (in terms of which Fermi and para-Fermi systems are described) does not commute with the Bose particle number operator. Therefore, if the Bose description of a fermion [such as that described in Section 3 of (A)] applies at a certain instant, then at a later stage it will no longer be correct. According to Section 4 of (A) the particle will be looked upon in the future as a parafermion with a certain probability for each order of parastatistics. The reason for this is that at the initial time when the system was looked upon as a fermion, this was because the state belonged to the one-boson subspace $\mathscr{B}_1$. In the future the state will generally still have a nonzero component in the original subspace $\mathscr{B}_1$; however, this component will usually be smaller and smaller as time evolves. The conclusion is the following: What we have is a picture of an unstable fermion resulting from the fact that initially the particle was a particular fermion; however, as time goes by the probability of changing from the original fermion to another particle becomes very high. *Thus, the Bose description of fermions and parafermions offers a possible geometrical picture as well as an eventual physical understanding of Fermi (and similarly para-Fermi) unstable particles.*

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